Weak decays of heavy hadron molecules involving the $\mathbf{f_0(980)}$

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We study weak decays of the charm- and bottom-strange mesons $D_{s0}^*(2317)$, $D_{s1}(2460)$, $B_{s0}^*(5725)$ and $B_{s1}(5778)$ with $f_0(980)$ in the final state by assuming a hadronic molecule interpretation for their structures. Since in the proposed framework the initial and final states are occupied by hadronic molecules, the predictions for observables can provide a sensitive tool to further test the hadronic molecule structure in future experiments.

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I. INTRODUCTION

Over the last decades it became clear that the meson mass spectrum shows a much richer structure than one might expect from the conventional constituent quark model assigning mesons as $q\bar{q}$ states. For example, the structure of the light scalar mesons below 1 GeV such as the $f_0(980)$ have been in the focus. The strong and electromagnetic decay properties of the scalar f_0 have been intensely studied in various models ranging from quarkonium and hybrid structures to compact tetraquarks and hadronic molecules (for overview see e.g. Ref. [1]).

Newer experiments delivering data in the heavier mass region also attracted interest on mesons with open and hidden charm flavor configurations. Within this context one has to mention the $D^*_{s0}(2317)$ which has the favored spin-parity assignments $J^P = 0^+$ and which was first observed by *BABAR* at SLAC [2]. Shortly afterwards the CLEO collaboration [3] published their data on the axial $D_{s1}(2460)$. Both resonances have been confirmed by Belle [4]. Up to now the structure issue of the $D^*_{s0}(2317)$ and $D_{s1}(2460)$ remains an open question. Both mesons have therefore been discussed within various structure assumptions and theoretical frameworks [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34].

Since their masses are located slightly below the DK and D^*K thresholds, the D^*_{s0} and D_{s1} mesons are clear candidates for hadronic molecules with the configurations $D^*_{s0}(2317) = DK$ and $D_{s1}(2460) = D^*K$. In addition, extending this interpretation to the bottom sector, the scalar and axial-vector mesons $B^*_{s0}(5725)$ and $B_{s1}(5778)$ are treated as the equivalents to the charm-strange mesons $D^*_{s0}(2317)$ and $D_{s1}(2460)$. The bottom-strange counterparts $B^*_{s0}(5725)$ and $B_{s1}(5778)$ are consequently also described as bound states with $B^*_{s0}(5725) = B\bar{K}$ and $B_{s1}(5778) = D^*K$. The decay properties of these hadronic molecules were studied within the same effective Lagrangian approach [31, 32, 33, 34, 35, 36, 37]. Within this covariant model for hadronic bound states, the molecular structure is considered by the compositeness condition $Z = 0$ [31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43] which implies that the renormalization constant of the hadronic molecule field is set equal to zero. The composite object therefore exists exclusively as a bound state of its constituents. This condition also provides a method to fix the coupling between the hadronic molecule and its constituent mesons in a self-consistent way. Furthermore, our theoretical framework also features finite size effects of the meson molecules controlled by size parameters which are the only adaptive variables.

In the present paper the $f_0(980)$ properties are studied in weak hadronic decays of the scalar $D^*_{s0}(2317)$ and its bottom-strange counterpart $B^*_{s0}(5725)$ as well as in the weak non-leptonic decay processes of the axial-vector mesons $D_{s1}(2460)$ and $B_{s1}(5778)$. Since we deal with transition processes between hadronic molecules, the decay properties involve twice the effect of meson bound states: In the initial heavy meson system and in the final scalar f_0. For this reason the results might provide a sensitive observable to test the issue of hadronic molecule structure accessible in future experiments.

The paper is organized as follows. In the next section II we give a short introduction to the effective Lagrangian approach we use for the description of hadronic bound states. In section III we deal with the weak non-leptonic decays of the scalar mesons $D^*_{s0}(2317)$ and $B^*_{s0}(5725)$, where the meson molecule f_0 appears in the final state. The $D^*K\pi$ coupling g_π, which we need for the $D^+_{s1}(2460) \to f_0\pi^+$ transition, is derived in Sec. III from the $D_s \to \pi f_0$ decay. Thereby we also obtain the $D^* \to K\pi$ decay width as a byproduct of our analysis. In Sec. IV we finally compute the f_0-production in hadronic decays of the axial-vector mesons $D_{s1}(2460)$ and $B_{s1}(5778)$.

II. BASICS OF THE MODEL

An assortment of mesons with masses lying close to two-body thresholds are good candidates for mesonic bound states and have therefore been studied assuming a hadronic molecule structure. For instance, in Refs. [31, 32, 33, 34, 35, 36, 37] we developed a field-theoretical approach to study the properties of hadronic molecules ($f_0(980)$, $D^*_{s0}(2317)$, $D_{s1}(2460)$, $B^*_{s0}(5725)$, $B_{s1}(5778)$ and $X(3872)$) as bound states of two mesons. Since above states are close to the

corresponding thresholds, we used the following dominant composite structures:

$$
\begin{aligned}
|f_0\rangle &= \frac{1}{\sqrt{2}}\big(|K^+K^-\rangle + |K^0\bar{K}^0\rangle\big)\,, \\
|D_{s0}^{*\,+}\rangle &= \frac{1}{\sqrt{2}}\big(|D^+K^0\rangle + |D^0K^+\rangle\big)\,, \\
|D_{s1}^{+}\rangle &= \frac{1}{\sqrt{2}}\big(|D^{*\,+}K^0\rangle + |D^{*\,0}K^+\rangle\big)\,, \\
|B_{s0}^{*\,0}\rangle &= \frac{1}{\sqrt{2}}\big(|B^+K^-\rangle + |B^0\bar{K}^0\rangle\big)\,, \\
|B_{s1}^{0}\rangle &= \frac{1}{\sqrt{2}}\big(|B^{*\,+}K^-\rangle + |B^{*\,0}\bar{K}^0\rangle\big)\,.
\end{aligned}
\tag{1}
$$

The model for hadronic molecules $H = f_0(980)$, $D_{s0}^*(2317)$, $D_{s1}(2460)$, $B_{s0}^*(5725)$ composed of two meson constituents M_1 and M_2 is thereby based on the nonlocal interaction Lagrangians

$$
\mathcal{L}_{HM_1M_2} = g_H H(x) \int dy\, \Phi_H(y^2)\, M_1^T(x + w_{21}y)\, M_2(x - w_{12}y) + \text{H.c.}\,, \tag{2}
$$

where M_1 and M_2 are the doublets of the meson fields:

$$
K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}, \quad D = \begin{pmatrix} D^0 \\ D^+ \end{pmatrix}, \quad D_\mu^* = \begin{pmatrix} D^{*\,0} \\ D^{*\,+} \end{pmatrix}_\mu, \quad B = \begin{pmatrix} B^+ \\ B^0 \end{pmatrix}, \quad B_\mu^* = \begin{pmatrix} B^{*\,+} \\ B^{*\,0} \end{pmatrix}_\mu \tag{3}
$$

and their antiparticles. The symbol T refers to the transpose of M_1. The kinematic variable w_{ij} is defined by $w_{ij} = m_i/(m_i + m_j)$ where m_1 and m_2 are the masses of M_1 and M_2.

The finite size of the hadronic molecule is introduced through the correlation function $\Phi_H(y^2)$ which describes the distribution of the constituent mesons. Its Fourier transform $\widetilde{\Phi}_H(k_E^2)$ appears as the form factor in our calculations, where, in the present analysis, we have chosen a Gaussian form

$$
\widetilde{\Phi}_H(k_E^2) = \exp(-k_E^2/\Lambda_H^2) \tag{4}
$$

in Euclidean momentum space. The size parameter Λ_H controls the spatial extension of the hadronic molecule and is varied between 1 - 2 GeV. The local case (LC), describing point-like interaction, is defined for $\Lambda_H \to \infty$. (Note this limit can be applied to convergent matrix elements only). The size parameters Λ_H are the only adjustable parameters in our framework.

The coupling constants between the hadronic molecules and its building blocks, the constituent mesons, are fixed self-consistently by the compositeness condition [31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43]. The dynamics of the bound state is therefore related to its constituents by setting the field renormalization constant to zero. Because of this constraint, the coupling constants are no input parameters but are fixed within this theoretical framework. The number of free variables is therefore reduced to the size parameters Λ_H. For the generic hadronic molecule $H = (M_1 M_2)$, the compositeness condition is given by the relation

$$
Z_H = 1 - \Sigma_H'(m_H^2) = 0\,, \tag{5}
$$

where $\Sigma_H'(m_H^2) = g_H^2 \Pi_H'(m_H^2)$ is the derivative of the mass operator (see Fig. 1) and m_H is the mass of hadronic molecule.

In the mesonic molecule picture all decays proceed via intermediate states which are the composite mesons of the hadronic bound state. We describe the dynamics of the intermediate states by free propagators given by the standard expressions

$$
iS_M(x-y) = \langle 0|TM(x)M^\dagger(y)|0\rangle = \int \frac{d^4k}{(2\pi)^4 i}\, e^{-ik(x-y)} S_M(k), \quad S_M(k) = \frac{1}{m_M^2 - k^2 - i\epsilon} \tag{6}
$$

for pseudoscalar and scalar fields M and by

$$
iS_{M^*}^{\mu\nu}(x-y) = \langle 0|TM^{*\,\mu}(x)M^{*\,\nu\,\dagger}(y)|0\rangle = \int \frac{d^4k}{(2\pi)^4 i}\, e^{-ik(x-y)} S_{M^*}^{\mu\nu}(k)\,, \quad S_{M^*}^{\mu\nu}(k) = \frac{-g^{\mu\nu} + k^\mu k^\nu/m_{M^*}^2}{m_{M^*}^2 - k^2 - i\epsilon} \tag{7}
$$

in case of vector and axial-vector fields M^*.

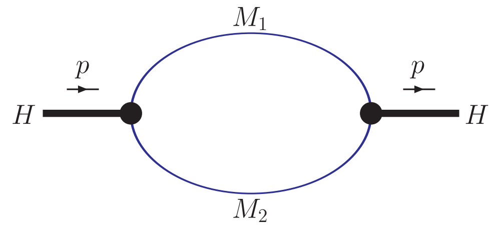

FIG. 1: Mass operator of the hadronic molecule.

For the D and B meson masses we use the the values quoted in [44] and estimated in [24]:

$$\begin{aligned}
&m_{D^+} = 1.8696 \text{ GeV}, \ \ m_{D^0} = 1.8648 \text{ GeV}, \ \ m_{D_s^+} = 1.96849 \text{ GeV}, \ \ m_{D^{*+}} = 2.01027 \text{ GeV}, \ \ m_{D^{*0}} = 2.00697 \text{ GeV}, \\
&m_{B^+} = 5.2791 \text{ GeV}, \ \ m_{B^0} = 5.2795 \text{ GeV}, \ \ m_{B^{*+}} = 5.3251 \text{ GeV}, \ \ m_{B^{*0}} = 5.3251 \text{ GeV}, \\
&m_{D^*_{s0}} = 2.3178 \text{ GeV}, \ \ m_{B^*_{s0}} = 5.725 \text{ GeV}, \ \ m_{D_{s1}} = 2.4596 \text{ GeV}, \ \ m_{B_{s_1}} = 5.778 \text{ GeV}.
\end{aligned} \tag{8}$$

Below we list our previous predictions for the couplings g_H obtained for the respective molecular states. In particular, for the $fK\bar{K}$-coupling we obtained [36]

$$g_{f_0} = 3.09 \text{ GeV} \quad (\Lambda_{f_0} = 1 \text{ GeV}) \,, \quad g_{f_0} = 2.9 \text{ GeV} \quad (\text{LC}) \,. \tag{9}$$

The coupling constants of the D^*_{s0} and D_{s1} mesons have already been calculated in [31, 32, 33]:

$$\begin{aligned}
&g_{D^*_{s0}} = 11.26 \text{ GeV} \quad (\Lambda_{D^*_{s0}} = 1 \text{ GeV}), \qquad g_{D^*_{s0}} = 9.9 \text{ GeV} \quad (\Lambda_{D^*_{s0}} = 2 \text{ GeV}), \qquad g_{D^*_{s0}} = 8.98 \text{ GeV} \quad (\text{LC}) \,, \\
&g_{D_{s1}} = 11.62 \text{ GeV} \quad (\Lambda_{D_{s1}} = 1 \text{ GeV}) \,, \qquad g_{D_{s1}} = 10.17 \text{ GeV} \quad (\Lambda_{D_{s1}} = 2 \text{ GeV}) \,.
\end{aligned} \tag{10}$$

The results for the couplings of the B^*_{s0} and B_{s1} mesons to their constituents for different size parameters Λ are [34]:

$$\begin{aligned}
&g_{B^*_{s0}} = 27.17 \text{ GeV} \quad (\Lambda_{B^*_{s0}} = 1 \text{ GeV}), \quad g_{B^*_{s0}} = 23.21 \text{ GeV} \quad (\Lambda_{B^*_{s0}} = 2 \text{ GeV}), \quad g_{B^*_{s0}} = 20.10 \text{ GeV} \quad (\text{LC}) \,, \\
&g_{B_{s1}} = 25.64 \text{ GeV} \quad (\Lambda_{B_{s1}} = 1 \text{ GeV}), \quad g_{B_{s1}} = 22.14 \text{ GeV} \quad (\Lambda_{B_{s1}} = 2 \text{ GeV}) \,.
\end{aligned} \tag{11}$$

One should stress that the coupling constants g_{f_0}, $g_{D^*_{s0}}$ and $g_{B^*_{s0}}$ of the scalar mesons f_0, D^*_{s0}, and B^*_{s0} remain finite when we remove the cutoff $\Lambda_H \to \infty$. For the axial mesons D_{s1} and B_{s1} the couplings $g_{D_{s1}}$ and $g_{B_{s1}}$ are finite in the local limit when we neglect the longitudinal part $k^\mu k^\nu / m^2_{M^*}$ of the constituent vector meson propagator. In this case all the couplings are given analytically by

$$\frac{1}{g_H^2} = \frac{2}{(4\pi m_H)^2} \left\{ \frac{m_1^2 - m_2^2}{m_H^2} \ln \frac{m_1}{m_2} - 1 + \frac{m_H^2(m_1^2 + m_2^2) - (m_1^2 - m_2^2)^2}{m_H^2 \sqrt{-\lambda}} \sum_{\pm} \arctan \frac{z_\pm}{\sqrt{-\lambda}} \right\} \tag{12}$$

where $z_\pm = m_H^2 \pm (m_1^2 - m_2^2)$ and

$$\lambda \doteq \lambda(m_H^2, m_1^2, m_2^2) = m_H^4 + m_1^4 + m_2^4 - 2m_H^2 m_1^2 - 2m_H^2 m_2^2 - 2m_1^2 m_2^2 \tag{13}$$

is the Källen function. When writing the mass m_H of the hadronic molecule in the form $m_H = m_1 + m_2 - \epsilon$, where ϵ represents the binding energy, we can perform an expansion of g_H^2 in powers of ϵ. The leading-order $\mathcal{O}(\sqrt{\epsilon})$ result

$$\frac{\mathring{g}_H^{\,2}}{4\pi} = \frac{(m_1 + m_2)^{5/2}}{\sqrt{m_1 m_2}} \sqrt{8\epsilon} \tag{14}$$

in agreement with the one derived in Refs. [38, 42, 43, 45] based on a formalism which also used the compositeness condition $Z_H = 0$.

Numerical results for the coupling constants $\mathring{g}_H$

$$\mathring{g}_{f_0} = 2.74 \text{ GeV} \,, \quad \mathring{g}_{D^*_{s0}} = 8.27 \text{ GeV} \,, \quad \mathring{g}_{D_{s1}} = 8.63 \text{ GeV} \,, \quad \mathring{g}_{B^*_{s0}} = 19.63 \text{ GeV} \,, \quad \mathring{g}_{B_{s1}} = 19.01 \text{ GeV} \,. \tag{15}$$

compare well with the results obtained in the local case without the ϵ expansion and in the nonlocal case (see Eqs. (9) and (10)). Note that in the calculation of $\mathring{g}_{f_0}$ we use the averaged kaon mass $\bar{m}_K = (m_{K^\pm} + m_{K^0})/2$.

For consistency we also analyze the couplings g_H and $\overset{\circ}{g}_H$ in the heavy quark limit (HQL), where the masses of the heavy mesons together with the heavy quark masses go to infinity. The scaling of the coupling constant $g_{D^*_{s0}}$ in the HQL was already discussed in [32]. It was shown that $g_{D^*_{s0}}$, both for the nonlocal and the local case, is proportional to the charm quark mass or the mass of the D^*_{s0} meson (see Eqs.(57) and (58) of Ref. [32]). This result is simply extended to the cases of the B^*_{s0} coupling and of the couplings of the axial states D_{s1} and B_{s1}. In particular, for the nonlocal case the result for g_H in the HQL is:

$$\frac{1}{g_H^2} \;=\; \frac{1}{(4\pi m_H)^2} \int\limits_0^\infty \frac{d\alpha}{1+\mu_K^2\alpha}\; \tilde\Phi_H^2(\alpha)\,, \tag{16}$$

where $\mu_K = m_K/\Lambda_H$. In the local case the HQL reads as:

$$\frac{1}{g_H^2} = \frac{1}{(4\pi m_H)^2} \ln\frac{m_H^2}{m_K^2}\,. \tag{17}$$

Hence, the coupling of the heavy-light molecules to the constituents is proportional to the heavy quark mass (or the molecule mass $m_H = m_Q + \mathcal{O}(1)$). Therefore, we deduce the following relations between the coupling constants g_H in the HQL:

$$\begin{aligned} & g_{D^*_{s0}} = g_{D_{s1}}\,, \quad g_{B^*_{s0}} = g_{B_{s1}}\,, \\ & \frac{g_{B^*_{s0}}}{g_{D^*_{s0}}} = \frac{g_{B_{s1}}}{g_{D_{s1}}} \simeq \frac{m_{B^*_{s0}}}{m_{D^*_{s0}}} \simeq \frac{m_{B_{s1}}}{m_{D_{s1}}}\,. \end{aligned} \tag{18}$$

This scaling behavior is also evident from Eq. (14), where the couplings $\overset{\circ}{g}_H$ behave in the HQL as:

$$\frac{\overset{\circ}{g}{}_H^2}{4\pi} = m_H^2\sqrt{\frac{8\epsilon}{m_K}}\,. \tag{19}$$

Keeping in mind that the binding energy ϵ is approximately the same for all four states (D^*_{s0}, B^*_{s0}, D_{s1}, B_{s1}), we deduce that in the HQL the relations (18) are also valid for the leading-order couplings $\overset{\circ}{g}_H$. Using the previous numerical values for the g_H and $\overset{\circ}{g}_H$ couplings one can see that the HQL relations (18) are fulfilled with a good accuracy. It also explains the phenomenon that the bottom meson couplings are 2.2 - 2.8 times larger than the charm ones.

III. $\mathbf{D^*_{s0}}$(2317) AND $\mathbf{B^*_{s0}}$(5725) DECAYS

In this section we deal with the f_0-production properties in weak hadronic decays of the heavy scalar mesons $D^*_{s0}(2317)$ and $B^*_{s0}(5725)$. Here the final states of the $D^{*+}_{s0} \to f_0 X$ decay are occupied by the charged mesons $X = \pi^+, K^+, \rho^+$ and the scalar f_0. The decay pattern of the neutral B^{*0}_{s0} decay is richer and we deal with final π^0, K^0, ρ^0, ω, η and η' mesons besides the f_0.

Since both heavy quark systems are assumed to be of molecular structure the decays proceed via intermediate kaons and D or B mesons as indicated in the diagrams of Figs. 2 and 3.

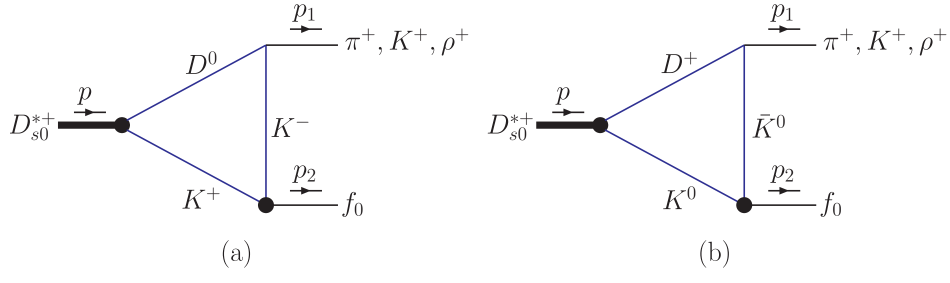

FIG. 2: Diagrams contributing to the $D^{*+}_{s0} \to f_0 X$ decays with $X = \pi^+$, K^+ and ρ^+.

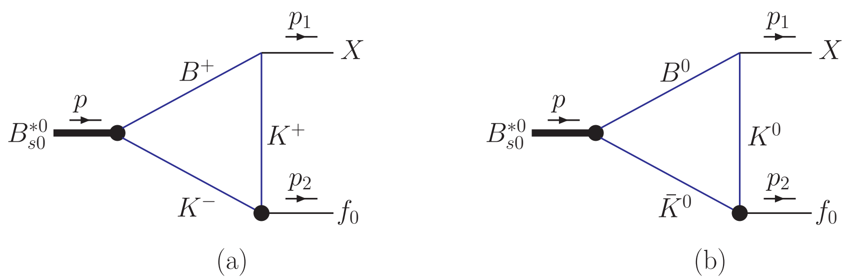

FIG. 3: Diagrams contributing to the $B_{s0}^{*0} \to f_0 X$ decays with $X = \pi^0,\ K^0,\ \rho^0,\ \omega,\ \eta$ and η'.

The couplings of the hadronic molecules to the constituent mesons in the loop are fixed by the compositeness condition. The coupling constants between the intermediate K, D and B mesons and the final decay products $\pi, K, \rho, \omega\,\eta$ and η' are obtained from the D and B meson partial decay widths. The latter constants are given by following expressions, where we distinguish between final pseudoscalar (P) and vector mesons (V):

$$g^{c(n)}_{HKP} \;=\; \sqrt{\frac{16\pi\,\Gamma(H\to K\,P)\,m_H^3}{\lambda^{\frac{1}{2}}(m_H^2,m_K^2,m_P^2)}}\,,\quad (P=K,\pi,\eta,\eta',\quad H=D,B)\,, \tag{20}$$

$$g^{c(n)}_{HKV} \;=\; \sqrt{\frac{64\pi\,\Gamma(H\to K\,V)\,m_H^3\,m_V^2}{\lambda^{\frac{3}{2}}(m_H^2,m_K^2,m_V^2)}}\,,\quad (V=\rho,\omega,\quad H=D,B)\,, \tag{21}$$

with the Källen function $\lambda(x,y,z)$ defined in Eq. (13). The superscript c (n) denotes the decays of the charged (neutral) D and B mesons.

The couplings governing the $D_{s0}^* \to f_0 P$ and $B_{s0}^* \to f_0 P$ decays we calculate from

$$g_{D_{s0}^* f_0\,P} \;=\; \frac{g_{D_{s0}^*}\,g_{f_0}}{(4\pi)^2}\big[g^c_{HKP}I(m_{D^+}^2,m_{K^0}^2)+g^n_{HKP}I(m_{D^0}^2,m_{K^+}^2)\big]\,, \tag{22}$$

$$g_{B_{s0}^* f_0\,P} \;=\; \frac{g_{B_{s0}^*}\,g_{f_0}}{(4\pi)^2}\big[g^c_{HKP}I(m_{D^+}^2,m_{K^+}^2)+g^n_{HKP}I(m_{D^0}^2,m_{K^0}^2)\big]\,, \tag{23}$$

where $I(m_H^2,m_K^2)$ denotes the loop integral

$$I(m_H^2,m_K^2) = \int\frac{d^4k}{\pi^2 i}\,\widetilde{\Phi}_{f_0}(-k^2)\widetilde{\Phi}_{H_{s0}^*}\big(-(k-\frac{p}{2}+\omega p_{H_{s0}^*})^2\big)\,S_H\big(k-\frac{p}{2}+p_{H_{s0}^*}\big)S_K\big(k-\frac{p}{2}\big)S_K\big(k+\frac{p}{2}\big)\,, \tag{24}$$

with $H_{s0}^* = B_{s0}^{*\,0}\,,D_{s0}^{*\,+}$.

The decay widths are finally obtained from

$$\Gamma(H_{s0}^*\to f_0\,P) \;=\; \frac{g^2_{H_{s0}^* f_0\,P}}{16\pi m^3_{H_{s0}^*}}\,\lambda^{\frac{1}{2}}(m^2_{H_{s0}^*},m^2_{f_0},m^2_P)\,. \tag{25}$$

For the decays with a final vector meson, $D_{s0}^*/B_{s0}^* \to f_0 V$, we proceed in analogy. For simplicity, we restrict in the following to the $D_{s0}^{*+} \to f_0\rho^+$ decay since the corresponding expressions for the bottom B_{s0}^* decays only differ in the masses and couplings, while the structure remains the same.

Again, the Feynman integral

$$\begin{aligned} I^\mu(m_D^2,m_K^2) \;&=\; \int\frac{d^4k}{\pi^2 i}\,\widetilde{\Phi}_{f_0}(-k^2)\,\widetilde{\Phi}_{D_{s0}^*}\big(-(k-\frac{p}{2}+\omega p_{D_{s0}^*})^2\big)\,(2k+p_{D_{s0}^*})^\mu \\ &\times\; S_D\big(k-\frac{p}{2}+p_{D_{s0}^*}\big)S_K\big(k-\frac{p}{2}\big)S_K\big(k+\frac{p}{2}\big) \end{aligned} \tag{26}$$

defines the transition matrix element $\mathcal{M}^\mu$ which is given by

$$\begin{aligned} \mathcal{M}^\mu \;&=\; \frac{g_{D_{s0}^*}g_{f_0}}{(4\pi)^2}\big[g^c_{HK\rho}I^\mu(m_{D^+}^2,m_{K^0}^2)+g^n_{HK\rho}I^\mu(m_{D^0}^2,m_{K^+}^2)\big] \\ &=\; F_1(m^2_{D_{s0}^*},m^2_{f_0},m^2_\rho)\,p_f^\mu+F_2(m^2_{D_{s0}^*},m^2_{f_0},m^2_\rho)\,p_\rho^\mu\,. \end{aligned} \tag{27}$$

In the second line $\mathcal{M}^\mu$ is expressed in terms of the form factors F_1 and F_2 by writing the matrix element as a linear combination of the f_0 and ρ meson momenta p_f and p_ρ. We perform this decomposition since the form factor F_1 defines the coupling constant of the decay

$$F_1(m^2_{D_{s0}}, m^2_{f_0}, m^2_\rho) \equiv g_{D^*_{s0} f_0 \rho} \tag{28}$$

and therefore characterizes the decay width with

$$\Gamma(D^{*+}_{s0} \to f_0 \rho^+) = \frac{g^2_{D^*_{s0} f_0 \rho}}{64 \pi m^3_{D^*_{s0}} m^2_\rho} \, \lambda^{\frac{3}{2}}(m^2_{D^*_{s0}}, m^2_{f_0}, m^2_\rho)\,. \tag{29}$$

First we indicate the results for the coupling constants at the secondary interaction vertex as deduced from the decays $B/D \to KX$ ($X = \pi,\ K, \eta', \eta, \omega, \rho$). In Table I we summarize the branching ratios (Br) as taken from and the resulting couplings $g_X^{c(n)}$ (via Eqs. (20) and (21)) involving charged (c) and neutral (n) B and D mesons.

TABLE I: Coupling constants deduced from the decays $B/D \to KX$ with $X = \pi,\ K,\ \eta',\ \eta,\ \omega,\ \rho$.

Channel	Br [44, 46]	g_X^n	Channel	Br [44, 46]	g_X^c
$D^0 \to \pi^+ K^-$	$(3.89 \pm 0.05)\,\%$	$2.88 \cdot 10^{-6}$ GeV	$D^+ \to \pi^+ \bar{K^0}$	$(2.83 \pm 0.18)\,\%$	$0.14 \cdot 10^{-5}$ GeV
$D^0 \to K^+ K^-$	$(3.93 \pm 0.08) \cdot 10^{-3}$	$0.83 \cdot 10^{-6}$ GeV	$D^+ \to K^+ \bar{K^0}$	$(5.7 \pm 0.5) \cdot 10^{-3}$	$0.63 \cdot 10^{-6}$ GeV
$D^0 \to \rho^+ K^-$	$(10.8 \pm 0.7)\,\%$	$2.92 \cdot 10^{-6}$	$D^+ \to \rho^+ \bar{K^0}$	$(7.3 \pm 2.5)\,\%$	$0.15 \cdot 10^{-5}$
$B^0 \to K^0 \pi^0$	$(9.8 \pm 0.6) \cdot 10^{-6}$	$3.36 \cdot 10^{-8}$ GeV	$B^+ \to K^+ \pi^0$	$(1.29 \pm 0.06) \cdot 10^{-5}$	$3.73 \cdot 10^{-8}$ GeV
$B^0 \to K^0 \eta'$	$(6.5 \pm 0.4) \cdot 10^{-5}$	$0.91 \cdot 10^{-7}$ GeV	$B^+ \to K^+ \eta'$	$(7.02 \pm 0.25) \cdot 10^{-5}$	$8.84 \cdot 10^{-8}$ GeV
$B^0 \to K^0 \eta$	$< 1.9 \cdot 10^{-6}$	$< 0.15 \cdot 10^{-7}$ GeV	$B^+ \to K^+ \eta$	$(2.7 \pm 0.9) \cdot 10^{-6}$	$0.17 \cdot 10^{-7}$ GeV
$B^0 \to K^0 \bar{K^0}$	$(9.6^{+2.0}_{-1.8}) \cdot 10^{-7}$	$1.06 \cdot 10^{-8}$ GeV	$B^+ \to K^+ \bar{K^0}$	$(1.36 \pm 0.27) \cdot 10^{-6}$	$1.22 \cdot 10^{-8}$ GeV
$B^0 \to K^0 \omega$	$(5.0 \pm 0.6) \cdot 10^{-6}$	$1.41 \cdot 10^{-9}$	$B^+ \to K^+ \omega$	$(6.7 \pm 0.8) \cdot 10^{-6}$	$1.57 \cdot 10^{-9}$
$B^0 \to K^0 \rho^0$	$(5.4 \pm 0.9) \cdot 10^{-6}$	$0.14 \cdot 10^{-8}$	$B^+ \to K^+ \rho^0$	$(4.2 \pm 0.5) \cdot 10^{-6}$	$1.23 \cdot 10^{-9}$

In Tables II and III we summarize the results for the coupling constants and decay widths of the $D^{*+}_{s0}(2317)$ and $B^{*0}_{s0}(5725)$ decays. We also indicate the dependence of the results for different sets of size parameters Λ_H. Compared to the local case (LC) finite size effects induce a reduction of the D^*_{s0} decay widths by up to 50%. For the B^*_{s0} decays inclusion of finite size parameters leads to a reduction of the partial decay widths by up to a factor of 10.

For the D^{*+}_{s0} decays we predict a decay pattern with

$$\Gamma(f_0 \rho^+) > \Gamma(f_0 \pi) > \Gamma(f_0 K^+)\,, \tag{30}$$

where the decay width of each sequential decay mode is reduced by about an order of magnitude. Here we introduce the shortened notation $\Gamma(D^{*+}_{s0} \to H_1 H_2) = \Gamma(H_1 H_2)$. In the case of B^{*0}_{s0} the weak decay mode $B^{*0}_{s0} \to f_0 \eta'$ dominates the transitions with the decay hierarchy

$$\Gamma(f_0 \eta') > \Gamma(f_0 \pi) \approx \Gamma(f_0 \rho) \approx \Gamma(f_0 \omega) > \Gamma(f_0 K) \approx \Gamma(f_0 \eta)\,. \tag{31}$$

TABLE II: $D^{*+}_{s0} \to f_0 X$ decay properties with $X = \pi^+,\ K^+,\ \rho^+$.

	$D^{*+}_{s0} \to f_0 \pi^+$		$D^{*+}_{s0} \to f_0 K^+$		$D^{*+}_{s0} \to f_0 \rho^+$	
Λ_H [GeV]	$g_{D^*_{s0} f_0 \pi}$ [GeV]	Γ [GeV]	$g_{D^*_{s0} f_0 K}$ [GeV]	Γ [GeV]	$g_{D^*_{s0} f_0 \rho}$	Γ [GeV]
LC	$1.83 \cdot 10^{-6}$	$2.35 \cdot 10^{-14}$	$6.51 \cdot 10^{-7}$	$2.75 \cdot 10^{-15}$	$2.37 \cdot 10^{-6}$	$1.60 \cdot 10^{-13}$
$\Lambda_{D^*_{s0}} = 2,\ \Lambda_{f_0} = 1$	$1.34 \cdot 10^{-6}$	$1.26 \cdot 10^{-14}$	$4.86 \cdot 10^{-7}$	$1.53 \cdot 10^{-15}$	$1.95 \cdot 10^{-6}$	$1.08 \cdot 10^{-13}$
$\Lambda_{D^*_{s0}} = 1, \Lambda_{f_0} = 1$	$1.28 \cdot 10^{-6}$	$1.14 \cdot 10^{-14}$	$4.68 \cdot 10^{-7}$	$1.42 \cdot 10^{-15}$	$1.98 \cdot 10^{-6}$	$1.11 \cdot 10^{-13}$

TABLE III: Results for $B_{s0}^{*0} \to f_0 X$ decays with $X = \pi^0,\ \eta',\ \eta,\ K^0,\ \omega,\ \rho^0$.

	local limit		$\Lambda_{B_{s0}^*} = 2$ GeV, $\Lambda_{f_0} = 1$ GeV		$\Lambda_{B_{s0}^*} = 1$ GeV, $\Lambda_{f_0} = 1$ GeV	
Channel	$g_{B_{s0}^* f_0 X}$	$\Gamma_{B_{s0}^* \to f_0 X}$ [GeV]	$g_{B_{s0}^* f_0 X}$	$\Gamma_{B_{s0}^* \to f_0 X}$ [GeV]	$g_{B_{s0}^* f_0 X}$	$\Gamma_{B_{s0}^* \to f_0 X}$ [GeV]
$B_{s0}^{*0} \to f_0 \pi^0$	$1.30 \cdot 10^{-8}$ GeV	$5.66 \cdot 10^{-19}$	$5.43 \cdot 10^{-9}$ GeV	$9.93 \cdot 10^{-20}$	$3.86 \cdot 10^{-9}$ GeV	$5.03 \cdot 10^{-20}$
$B_{s0}^{*0} \to f_0 \eta'$	$3.35 \cdot 10^{-8}$ GeV	$3.67 \cdot 10^{-18}$	$1.43 \cdot 10^{-8}$ GeV	$6.69 \cdot 10^{-19}$	$1.03 \cdot 10^{-8}$ GeV	$3.49 \cdot 10^{-19}$
$B_{s0}^{*0} \to f_0 \eta$	$< 5.89 \cdot 10^{-9}$ GeV	$< 1.16 \cdot 10^{-19}$	$< 2.48 \cdot 10^{-9}$ GeV	$< 2.05 \cdot 10^{-20}$	$< 1.77 \cdot 10^{-9}$ GeV	$< 1.05 \cdot 10^{-20}$
$B_{s0}^{*0} \to f_0 K^0$	$4.19 \cdot 10^{-9}$ GeV	$5.88 \cdot 10^{-20}$	$1.77 \cdot 10^{-9}$ GeV	$1.04 \cdot 10^{-20}$	$1.26 \cdot 10^{-9}$ GeV	$5.32 \cdot 10^{-21}$
$B_{s0}^{*0} \to f_0 \rho^0$	$5.89 \cdot 10^{-10}$	$4.64 \cdot 10^{-19}$	$2.63 \cdot 10^{-10}$	$9.22 \cdot 10^{-20}$	$2.08 \cdot 10^{-10}$	$5.75 \cdot 10^{-20}$
$B_{s0}^{*0} \to f_0 \omega$	$6.69 \cdot 10^{-10}$	$5.86 \cdot 10^{-19}$	$2.99 \cdot 10^{-10}$	$1.17 \cdot 10^{-19}$	$2.36 \cdot 10^{-10}$	$7.31 \cdot 10^{-20}$

IV. $\mathbf{D_s^+ \to f_0 \pi^+}$ decay

In this section we analyze the $D_s^+ \to f_0 \pi^+$ decay in order to derive a value for the $D^* K \pi$ coupling constant g_π. This coupling is needed for the calculation of the $D_{s1} \to f_0 \pi$ decay width discussed in the next section. In this context we also obtain the decay width $\Gamma(D^* \to K\pi)$ as an additional result. The D_s-decay is illustrated by the Feynman

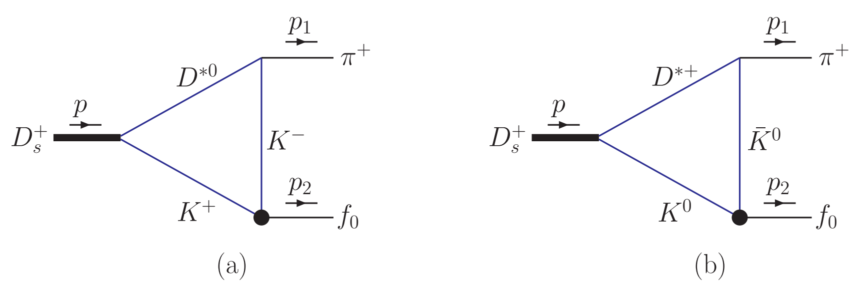

FIG. 4: D_s-decay.

diagrams of Fig. 4, where the decay width is defined as

$$\Gamma(D_s^+ \to f_0 \pi^+) = \frac{g^2_{D_s f_0 \pi}}{16\,\pi\, m^3_{D_s}}\, \lambda^{\frac{1}{2}}(m^2_{D_s}, m^2_{f_0}, m^2_\pi)\,. \tag{32}$$

The decay coupling

$$g_{D_s f_0 \pi} \;=\; \frac{g_f g_{D_s} g_\pi}{(4\pi)^2}\left[I(m^2_{D^{*+}}, m^2_{K^0}) + I(m^2_{D^{*0}}, m^2_{K^+})\right] \tag{33}$$

can be computed from the loop integral $I(m^2_{D^*}, m^2_K)$ given by

$$I(m^2_{D^*}, m^2_K) \;=\; \int \frac{d^4k}{\pi^2 i}\, \widetilde{\Phi}_f(-k^2)\, \Big(p_\pi - k - \frac{p}{2}\Big)_\mu \Big(k - \frac{p}{2} - p_{D_s}\Big)_\nu\, S_D^{\mu\nu}\Big(k - \frac{p}{2} + p_{D_s}\Big) S_K\Big(k - \frac{p}{2}\Big) S_K\Big(k + \frac{p}{2}\Big)\,. \tag{34}$$

The coupling constant g_{D_s} of the $D_s D^* K$ interaction vertex has been estimated in two different QCD sum rule approaches [47, 48], where both results do not vary significantly from each other. Here we use the result of the QCD sum rule approach in [47] with $g_{D_s} = 2.02$. By using the branching ratio $\mathrm{Br}(D_s^+ \to f_0 \pi^+) = (6.0 \pm 2.4) \cdot 10^{-3}$ [44], corresponding to $\Gamma(D_s^+ \to f_0 \pi^+) = 7.9 \cdot 10^{-15}$ GeV, g_π can be easily derived from (32) and (33):

$$g_\pi = 6.41 \cdot 10^{-5}\,. \tag{35}$$

Now, the $D^* \to K\pi$ decay width is immediately given by

$$\Gamma(D^* \to K\pi) = \frac{g_\pi^2}{48\pi m^5_{D^*}}\, \lambda^{\frac{3}{2}}(m^2_{D^*}, m^2_K . m^2_\pi) \tag{36}$$

which leads to $\Gamma(D^* \to K\pi) = 4.45 \cdot 10^{-11}$ GeV.

V. $\mathbf{D_{s1}(2460)}$ AND $\mathbf{B_{s1}(5778)}$ DECAYS

In this section we study the properties of the weak transitions between the axial vector hadronic molecules $D_{s1}(2460)$ and $B_{s1}(5778)$ and the scalar $f_0(980)$. The determination of g_π in the last section enables us to compute the decay $D^+_{s1}(2460) \to f_0\pi^+$ within the $K\ D^*$ bound state framework. The Feynman diagrams which contribute to this decay are illustrated in Fig. 5. In the first step we define the matrix element of the $D^+_{s1} \to f_0\pi^+$ transition in terms of the

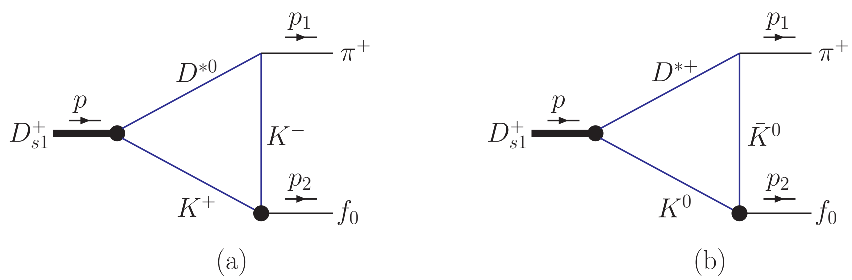

FIG. 5: $D^*_{s1}(2460)$ decay.

form factors $F_\pm$ and $p_\pm = p_f \pm p_\pi$

$$\begin{aligned}\mathcal{M}^\mu &= \frac{g_f g_{D_{s1}} g_\pi}{(4\pi)^2}\big(I^\mu(m^2_{D^{*+}}, m^2_{K^0}) + I^\mu(m^2_{D^{*0}}, m^2_{K^+})\big)\\ &= F_+(m_{D_{s1}}, m_\pi, m_{f_0})p^\mu_+ + F_-(m_{D_{s1}}, m_\pi, m_{f_0})p^\mu_- \,, \end{aligned} \tag{37}$$

where p_f and p_π are the f_0 and π momenta, respectively.

The loop integral involving the constituent kaons and D^* meson is of the structure

$$\begin{aligned} I^\mu(m^2_{D^*}, m^2_K) &= \int \frac{d^4k}{\pi^2 i}\,\widetilde{\Phi}_{f_0}(-k^2)\widetilde{\Phi}_{D_s}\big(-\big(k-\frac{p}{2}+\omega p_{D^*}\big)^2\big)\,\big(p_\pi - k - \frac{p}{2}\big)_\nu \\ &\times\ S^{\mu\nu}_{D^*}\big(k-\frac{p}{2}+p_{D_{s1}}\big)S_K\big(k-\frac{p}{2}\big)S_K\big(k+\frac{p}{2}\big)\,. \end{aligned} \tag{38}$$

The form factor F_- defines the coupling $g_{D_{s1}f_0\pi} = F_-(m_{D_{s1}}, m_\pi, m_{f_0})$ which characterizes the decay width given by the expression

$$\Gamma(D^+_{s1} \to f_0\pi^+) = \frac{g^2_{D_{s1}f_0\pi}}{48\pi\, m^5_{D^+_{s1}}}\,\lambda^{\frac{3}{2}}(m^2_{D^+_{s1}}, m^2_{f_0}, m^2_{\pi^+})\,. \tag{39}$$

We compute the decay width for $D^+_{s1} \to f_0\pi^+$ for the f_0 size parameter Λ_{f_0}=1 GeV while $\Lambda_{D_{s1}}$ is varied between 1 GeV and 2 GeV.

The results for the $D_{s1} \to f_0\pi$ decay width obtained within our hadronic molecule approach range from

$$\Gamma(D_{s1} \to f_0\pi) = 2.85\cdot 10^{-11}\ \text{GeV}, \quad \text{where} \quad g_{D_{s1}f_0\pi} = 5.46\cdot 10^{-5} \quad \text{at}\ \Lambda_{D_{s1}} = 1\ \text{GeV} \tag{40}$$

to

$$\Gamma(D_{s1} \to f_0\pi) = 4.35\cdot 10^{-11}\ \text{GeV}, \quad \text{where} \quad g_{D_{s1}f_0\pi} = 6.74\cdot 10^{-5} \quad \text{at}\ \Lambda_{D_{s1}} = 2\ \text{GeV}. \tag{41}$$

By analogy, we can also study the $B_{s1} \to f_0X$ decay, where P represents a pseudoscalar final state. However, since no data are available to determine the $B^* f_0 P$ coupling strength g_{B^*}, we quote the width and corresponding decay coupling in dependence on g_{B^*}. Varying $\Lambda_{B_{s1}}$ from 1.0 GeV to 2 GeV the width lies between

$$\Gamma(B_{s1} \to f_0\pi) = 8.82\cdot 10^{-6}\, g^2_{B^*}\ \text{GeV, where} \quad g_{B_{s1}f_0\pi} = 0.016\, g_{B^*} \quad \text{at}\ \Lambda_{B_{s1}} = 1\ \text{GeV} \tag{42}$$

and

$$\Gamma(B_{s1} \to f_0\pi) = 4.03\cdot 10^{-5}\, g^2_{B^*}\ \text{GeV, where} \quad g_{B_{s1}f_0\pi} = 0.034\, g_{B^*} \quad \text{at}\ \Lambda_{B_{s1}} = 2\ \text{GeV}. \tag{43}$$

VI. SUMMARY

In the present paper we focused on weak hadronic production processes of the scalar $f_0(980)$. For this purpose we studied the weak non-leptonic decays of the heavy mesons D_{s0}^{*+}, D_{s1}^+ as well as the B_{s0} and B_{s1} mesons assigned as the corresponding states in the bottom-strange sector.

The formalism presented provides a clear and straightforward method to study the issue of hadronic molecules. Since all coupling constants are either fixed self-consistently by the compositeness condition or are deduced from experimental data, the only adaptive variables are the size parameters of the meson molecules which allow for their extended structure. Finite size effects are studied by varying the size parameters within a physically reasonable region between 1 and 2 GeV. Additionally we also compare the results with finite size effects to the local case related to point-like interactions.

The molecular interpretation of both, the initial heavy mesons and the final decay product - the kaonic bound state f_0 - in the weak decays possibly offers a sensitive tool to study the structure issue. In particular for the $D_{s0}^*(2317) \to f_0 X$ transitions we give clear predictions for the decay pattern arising in the hadronic molecule picture, both for D_{s0}^* and f_0. Similarly, the result for the process $D_{s1} \to f_0\pi$ is a straightforward consequence of the molecular interpretation. In addition the $D^* \to f_0\pi$ decay properties can also be used to get information on the f_0 substructure.

Presently no comparative calculations, as for example in the full or partial quark-antiquark interpretation of the D_{s0}^{*+}, D_{s1}^+ and f_0 mesons, exist. Hence, the real sensitivity of the results for the weak processes studied here on details of the meson structure remains to be seen. But judging from previous model calculations of for example the dominant observed decay modes of the D_{s0}^* and D_{s1} a strong dependence on the structure models can be expected. Therefore, upcoming experiments measuring the weak production processes involving the scalar meson $f_0(980)$ could lead to new insights into the meson spectrum and its structure issue.

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